

Performance Evaluation of Kinetic Code on Scalar Processors

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Eulerian Kinetic (Vlasov) Simulations for Space Plasma Studies

Basic equations for collisionless space plasma:

- Maxwell equations (for electromagnetic wave propagations)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

Computational load less than 0.1%

- Collisionless Boltzmann equation with electromagnetic field (known as Vlasov equation for charged particle motions)

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

$$f(x, y, z, v_x, v_y, v_z)$$

6D! \Rightarrow 5D

Operator splitting into three equations [Umeda et al. 2012a]

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} = 0$$

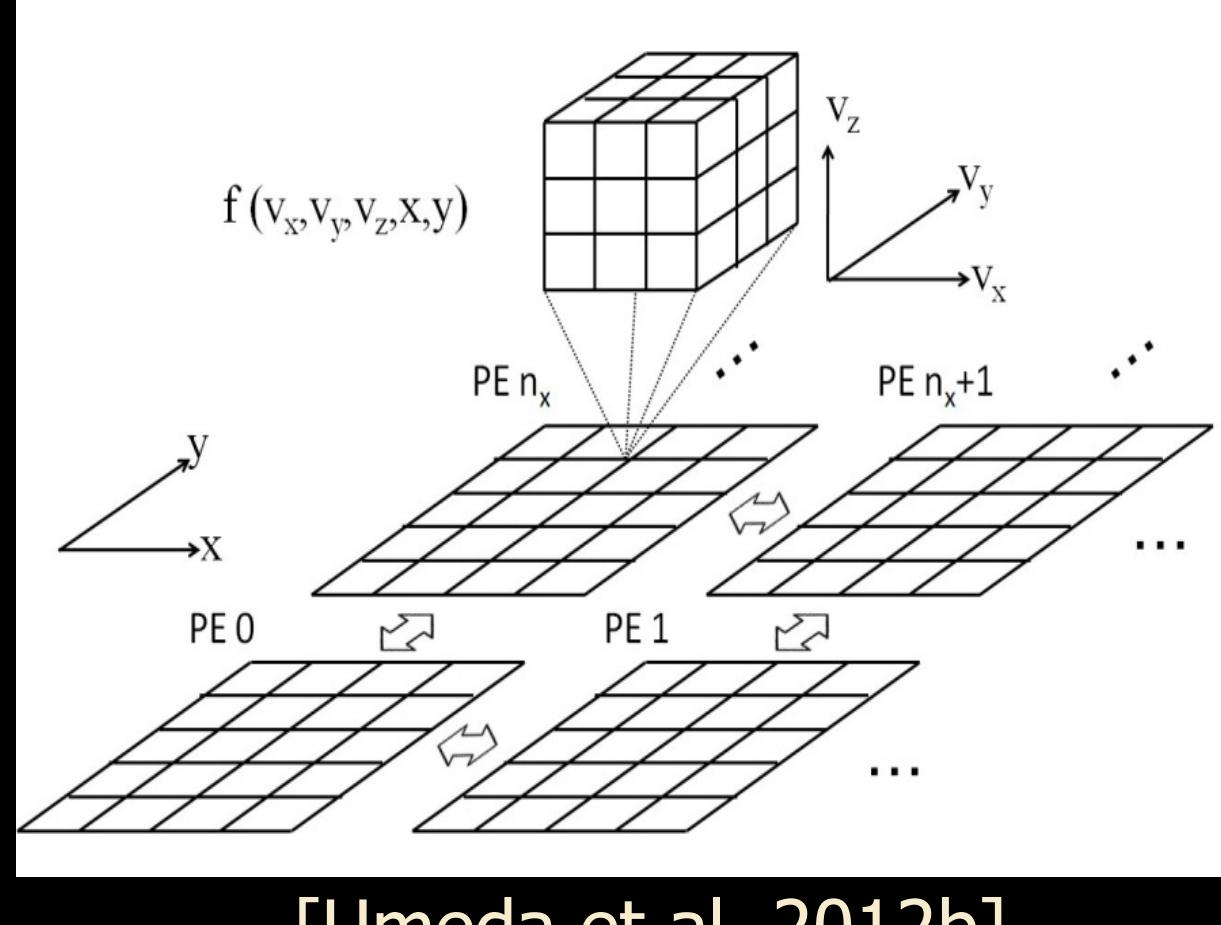
$$\frac{\partial f_s}{\partial t} + \frac{q_s}{m_s} \mathbf{E} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

$$\frac{\partial f_s}{\partial t} + \frac{q_s}{m_s} (\mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

(advection in position by \mathbf{v}) (advection in velocity by \mathbf{E}) (rotation by \mathbf{B})

Non-oscillatory, positive and conservative interpolation is used

[Umeda 2009; Umeda et al. 2012]



[Umeda et al. 2012b]

$40^5 \sim 4GB$

$40^6 \sim 160GB$

- Hybrid parallelism is adopted to reduce number of processes
- Large number of dimensions (up to 6) \Rightarrow Requires huge memory
- Length of each loop is short: 20-40 \Rightarrow number of threads > loop length in many core environments.
- Multiple loops are thread-parallelized by loop collapsing of OpenMP

[Umeda et al. 2016]

Performance Tuning

```

1 do n=0,nvx+1
2   do m=0,nvy+1
3     do l=0,nvz+1
4       ffi(l)=1.0d0/ff(l,m,n,i,j)
5     end do
6   end do
7
8   do l=0,nvx+1
9     gfyx = abs((gfy(l)*ffi(l))* gfy(l))           *0.5d0
10    gfyx = abs((gfy(l)*ffi(l))* gfxz(l))           *0.5d0
11    gfxz = abs((gfz(l)*ffi(l))* gfx(l))           *0.5d0
12    gfxz = abs((gfy(l)*ffi(l))*(gfy(l)*ffi(l))*gfxz(l))*inv3
13    dfx(l+10,m,n) = dfx(l+10,m,n) + (gfx(l)+(gfyx-gfxz-gfxz)*sx)
14    dfx(l+10,m,n+nv) = dfx(l+10,m,n+nv) - (gfyx-gfxz)*sx
15    dfx(l+10,m+mv,n) = dfx(l+10,m+mv,n) - (gfyx-gfxz)*sx
16    dfx(l+10,m+mv,n+nv) = dfx(l+10,m+mv,n+nv) + gfyx           *sx
17    dfy(l,m+m0,n) = dfy(l,m+m0,n) + (gfy(l)+(gfyx-gfxz-gfxz)*sy)
18    dfy(l+lv,m+m0,n) = dfy(l+lv,m+m0,n) - (gfyx-gfxz)*sy
19    dfy(l,m+m0,n+nv) = dfy(l,m+m0,n+nv) - (gfyx-gfxz)*sy
20    dfy(l+lv,m+m0,n+nv) = dfy(l+lv,m+m0,n+nv) + gfyx           *sy
21    dfz(l,m,n+n0) = dfz(l,m,n+n0) + (gfz(l)+(gfyx-gfxz-gfxz)*sz)
22    dfz(l,m+mv,n+n0) = dfz(l,m+mv,n+n0) - (gfyx-gfxz)*sz
23    dfz(l+lv,m,n+n0) = dfz(l+lv,m,n+n0) - (gfyx-gfxz)*sz
24    dfz(l+lv,m+mv,n+n0) = dfz(l+lv,m+mv,n+n0) + gfyx           *sz
25    dfz(l+lv,m+mv,n+n0) = dfz(l+lv,m+mv,n+n0) + gfyx           *sz
26
27  end do
28 end do
29 end do
30
31  gfyx = abs((gfy(l)*ffi(l))* gfy(l))           *0.5d0
32  gfyx = abs((gfy(l)*ffi(l))* gfxz(l))           *0.5d0
33  gfxz = abs((gfz(l)*ffi(l))* gfx(l))           *0.5d0
34  gfxz = abs((gfy(l)*ffi(l))*(gfy(l)*ffi(l))*gfxz(l))*inv3
35  dfx(l+10,m,n) = dfx(l+10,m,n) + (gfx(l)+(gfyx-gfxz-gfxz)*sx)
36  dfx(l+10,m,n+nv) = dfx(l+10,m,n+nv) - (gfyx-gfxz)*sx
37  dfx(l+10,m+mv,n) = dfx(l+10,m+mv,n) - (gfyx-gfxz)*sx
38  dfx(l+10,m+mv,n+nv) = dfx(l+10,m+mv,n+nv) + gfyx           *sx
39  dfy(l,m+m0,n) = dfy(l,m+m0,n) + (gfy(l)+(gfyx-gfxz-gfxz)*sy)
40  dfy(l+lv,m+m0,n) = dfy(l+lv,m+m0,n) - (gfyx-gfxz)*sy
41  dfy(l,m+m0,n+nv) = dfy(l,m+m0,n+nv) - (gfyx-gfxz)*sy
42  dfy(l+lv,m+m0,n+nv) = dfy(l+lv,m+m0,n+nv) + gfyx           *sy
43  dfz(l,m,n+n0) = dfz(l,m,n+n0) + (gfz(l)+(gfyx-gfxz-gfxz)*sz)
44  dfz(l,m+mv,n+n0) = dfz(l,m+mv,n+n0) - (gfyx-gfxz)*sz
45  dfz(l+lv,m,n+n0) = dfz(l+lv,m,n+n0) - (gfyx-gfxz)*sz
46  dfz(l+lv,m+mv,n+n0) = dfz(l+lv,m+mv,n+n0) + gfyx           *sz
47
48 end do
49 end do
50
51  do ll=nvx-1,nvx+1
52    ffi(l)=1.0d0/ff(ll,mm,nn,ii,jj)
53    gfx(ll) = abs(gfx(ll)*ffi)
54    gfy(ll) = abs(gfy(ll)*ffi)
55    gfz(ll) = abs(gfz(ll)*ffi)
56  end do
57
58  do ll=nvx-1,nvx+1
59    dfx(ll+10,mm,nn) = dfx(ll+10,mm,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*(gfy(ll)*(gfz(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sx
60    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*(gfz(ll)*(gfy(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sy
61    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) - ff(ll,mm,nn,ii,jj)*gfy(ll)*(gfz(ll)*(gfy(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sz
62
63  end do
64
65  do ll=nvx-1,nvx+1
66    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) - ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*(gfy(ll)*(gfz(ll)*inv3-0.5d0))*sx
67    dfy(ll,mm+m0,nn+nv) = dfy(ll,mm+m0,nn+nv) - ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*(gfy(ll)*inv3-0.5d0)*sy
68    dfz(ll,mm+m0,nn+nv) = dfz(ll,mm+m0,nn+nv) - ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*(gfy(ll)*inv3-0.5d0)*sz
69
70  end do
71
72  do ll=nvx-1,nvx+1
73    dfx(ll+10,mm+mv,nn+nv) = dfx(ll+10,mm+mv,nn+nv) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
74    dfy(ll,mm+m0,nn+nv) = dfy(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
75    dfz(ll,mm+m0,nn+nv) = dfz(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
76
77  end do
78
79  do ll=nvx-1,nvx+1
80    dfx(ll+10,mm+mv,nn+nv) = dfx(ll+10,mm+mv,nn+nv) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
81    dfy(ll,mm+m0,nn+nv) = dfy(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
82    dfz(ll,mm+m0,nn+nv) = dfz(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
83
84  end do
85
86  do ll=nvx-1,nvx+1
87    dfx(ll+10,mm+mv,nn+nv) = dfx(ll+10,mm+mv,nn+nv) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
88    dfy(ll,mm+m0,nn+nv) = dfy(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
89    dfz(ll,mm+m0,nn+nv) = dfz(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
90
91  end do
92
93  do ll=nvx-1,nvx+1
94    dfx(ll+10,mm+mv,nn+nv) = dfx(ll+10,mm+mv,nn+nv) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
95    dfy(ll,mm+m0,nn+nv) = dfy(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
96    dfz(ll,mm+m0,nn+nv) = dfz(ll,mm+m0,nn+nv) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
97
98  end do
99
100 end do
101
102 do ll=nvx-1,nvx+1
103   ffi(l)=1.0d0/ff(ll,mm,nn,ii,jj)
104   gfx(ll) = abs(gfx(ll)*ffi)
105   gfy(ll) = abs(gfy(ll)*ffi)
106   gfz(ll) = abs(gfz(ll)*ffi)
107 end do
108
109 do ll=nvx-1,nvx+1
110   dfx(ll+10,mm,nn) = dfx(ll+10,mm,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*(gfy(ll)*(gfz(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sx
111   dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*(gfz(ll)*(gfy(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sy
112   dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) - ff(ll,mm,nn,ii,jj)*gfy(ll)*(gfz(ll)*(gfy(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sz
113
114 end do
115
116  do ll=nvx-1,nvx+1
117    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) - ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*(gfy(ll)*(gfz(ll)*inv3-0.5d0)+(1.0d0-gfx(ll)*0.5d0))*sx
118    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) - ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*(gfy(ll)*inv3-0.5d0)*sy
119    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) - ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*(gfy(ll)*inv3-0.5d0)*sz
120
121  end do
122
123  do ll=nvx-1,nvx+1
124    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
125    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
126    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
127
128  end do
129
130  do ll=nvx-1,nvx+1
131    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
132    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
133    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
134
135  end do
136
137  do ll=nvx-1,nvx+1
138    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
139    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
140    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
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142  end do
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144  do ll=nvx-1,nvx+1
145    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
146    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
147    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
148
149  end do
150
151  do ll=nvx-1,nvx+1
152    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
153    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
154    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
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156  end do
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158  do ll=nvx-1,nvx+1
159    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
160    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
161    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
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163  end do
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165  do ll=nvx-1,nvx+1
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167    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
168    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
169
170  end do
171
172  do ll=nvx-1,nvx+1
173    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
174    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
175    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
176
177  end do
178
179  do ll=nvx-1,nvx+1
180    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
181    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
182    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
183
184  end do
185
186  do ll=nvx-1,nvx+1
187    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
188    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
189    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
190
191  end do
192
193  do ll=nvx-1,nvx+1
194    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
195    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
196    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
197
198  end do
199
200  do ll=nvx-1,nvx+1
201    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
202    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
203    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
204
205  end do
206
207  do ll=nvx-1,nvx+1
208    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
209    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sy
210    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
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212  end do
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214  do ll=nvx-1,nvx+1
215    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
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217    dfz(ll,mm+m0,nn) = dfz(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll)*gfz(ll)*inv3*sz
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219  end do
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222    dfx(ll+10,mm+mv,nn) = dfx(ll+10,mm+mv,nn) + ff(ll,mm,nn,ii,jj)*gfx(ll)*gfy(ll)*gfz(ll)*inv3*sx
223    dfy(ll,mm+m0,nn) = dfy(ll,mm+m0,nn) + ff(ll,mm,nn,ii,jj)*gfy(ll)*gfx(ll
```